SUMMARY PAGE FOR ALL LAB REPORTS

Laboratory exercises for the core course are generally designed to demonstrate or verify concepts presented in class. This goal is very distinct from that of experiments by research scientists that are designed to produce new knowledge and understanding. Your overall guide in preparing a lab report should be the goal of observing or verifying the concepts presented in lecture. You comments, figure captions, ... should be crafted to convince your instructor that you achieved the goal of the exercise.

A second issue is to assess your confidence in the result. "My measurements confirm that the resistance remained constant over the range of currents and voltages investigated to within 0.04 %." A careful statement of one's confidence in a result requires that one propagate uncertainties as discussed in the uncertainties handout.

Preface each report with a summary page that briefly tells:

I. What you learned from the exercise. This item should include specifics!

II. What you did beyond the instructions or instead of the instructions to get good results.

III. Changes that you would propose to the exercise or to the printed instructions.

IV The printed name and SIGNATURE of each student expecting a grade from the report. The signature indicates that the individual approves the report and submits for his/her grade.

Every plot will have a title and a short caption describing the data displayed at least detailed enough to uniquely identify it. I appreciate thoughtful captions.

Whenever possible base your responses on your measurements and observations; reference your measured values. Next base your response on your model for the phenomena. Last: Appeal to a model of equation from class or the text.

Always use complete sentences. State your point clearly.

Do and report clever things: When the two current probes where inserted in the series circuit, they gave slightly different readings. Interchange them to show that the difference reflects a difference in calibration rather than a difference in the current at the two points in the circuit.

An *ideal* battery always maintains a constant terminal potential difference between its terminals. A physical battery suffers some droop in terminal potential difference as the current drawn through it increases. Model: $V_{terminal} = V_o - r\,I$ where r is a small constant value, the *internal resistance*. You noted this droop with the 22Ω resistor and certainly with the light bulb,

Charge flows around closed paths in circuits. For our examples, those paths were completed by the motion of charges inside the battery resulting in a transfer from one terminal to the other.

Submit a completed lab instruction form with the summary page stapled first and with plot printouts stapled last

Special question for lab 1. Measure the resistance of the light bulb using the Protek. The result will be a value significantly less than that measured in the LabPro setup. What was different about the bulb during the two measurement schemes?

METER UNCERTAINIES & UNCERTAINTY PROPAGATION

The Mastech and Protek meter have uncertainties for each scale. These uncertainties are quoted as a % of the reading plus a number of 'digits'. The % of the reading is clear enough, but what is a digit? A digit is a least count change in the reading on the scale in use. In the tables of accuracy and resolution in the meter manuals, the value of a 'digit' is listed as the **resolution**.

Mastech Examples:

VOLTAGE: Mastech Meter set to DC 2V scale and reading 1.5273 *V*.

The uncertainty spec for the 2 *V* range: ± [0.1% of rdg + 3 digits]; Resolution 0.1 *mV*.
This translates to: ± [0.001 * (1.5273) + 3 * (0.0001)] = ± 0.00183 *V*.

The value to report is (1.5273 ± 0.00183) *V* = (1.527 ± 0.002) *V*.

CAPACITANCE: Mastech Meter set to 2 μ*F* and reading 1.5273 μ*F*

The uncertainty spec for the 2 μ*F* range: ± [4.0% of rdg + 20 digits]; Resolution 0.1 *nF*.

This translates to: ± [0.04 * (1.5273μ*F*) + (0.0001 μ*F*) * 20]

$$=\pm[0.06109 \quad + 2.00 \times 10^{-3}\ \mu F\ ]= \pm\ 0.06309\ \ \mu F.$$

The value to report is (1.5273 ± 0.06309) μ*F* = (1.53 ± 0.06) μ*F*.

Using the Mastech Uncertainty Tables.

Uncertainty = % of rdg + (# of digits * scale resolution)

The Protek thinks for itself! It auto-ranges: ***it chooses*** the most sensitive range that can accommodate your measured value. The various range limits for the Protek are of the form 4×10^{n} whereas they are of the form 2×10^{n} for the Mastech.

VOLTAGE: Protek Meter in the DC 4 *V* range* and reading 2.527 *V*.

The uncertainty spec for the 4 *V* range: ± [0.5% of rdg + 2 digits]; Resolution 0.001 *V*
This translates to: ± [0.005 * (2.527) + (0.001 *V*) * 2] =
= (0.01264 ± 0.002) *V* = 0.015 *V*.

The value to report is (2.527 ± 0.015) *V*.

Uncertainty = % of rdg + (# of digits * scale resolution)
= ± [0.005 * (2.527 *V*) + 2 *(0.0001*V*)]

**** Beware: the Protek changes ranges as the value measured changes. Be sure you know the range that is being used. It should be the most sensitive range that contains the measured value. That is: a voltage 3.91V is measured using the 4 V scale while 4.27 V is measured using the 40 V scale. Two digits on the 40 V scale is 0.02 V rather than the trivial 0.002 V that equates to two digits on the 4 V scale.***

Propagating Uncertainties: Examples for capacitors in series and parallel.

Series: Each measurement yields an expected (bar) value with its associated uncertainty. $C_1 \to \overline{C}_1 \pm \Delta C_1$ and $C_2 \to \overline{C}_2 \pm \Delta C_2$

The next step is to propagate the uncertainties in the measured values to find the uncertainty in a derived or computed quantity. The main rules are:

RULE U1: You **add** absolute uncertainties when you add or subtract values.

RULE U2: You **add** relative uncertainties when you multiply or divide quantities.

In the expression $C_1 \to \overline{C}_1 \pm \Delta C_1$, ΔC_1 is an absolute uncertainty. Re-expressed, $C_1 \to \overline{C}_1 \pm \Delta C_1 \to \overline{C}_1\left[1 \pm {}^{\Delta C_1}\!/_{C_1}\right]$ where ${}^{\Delta C_1}\!/_{C_1}$ is the relative (fractional) uncertainty.

Predicting the capacitance for adding in series.

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \to \frac{\left(\overline{C}_1 \pm \Delta C_1\right)\left(\overline{C}_2 \pm \Delta C_2\right)}{\left(\overline{C}_1 \pm \Delta C_1\right) + \left(\overline{C}_2 \pm \Delta C_2\right)}$$

Apply U2 to the numerator and U1 to the denominator.

$$C_s \pm \Delta C_s \to \frac{\left(\overline{C}_1 \pm \Delta C_1\right)\left(\overline{C}_2 \pm \Delta C_2\right)}{\left(\overline{C}_1 \pm \Delta C_1\right) + \left(\overline{C}_2 \pm \Delta C_2\right)} = \frac{\left(\overline{C}_1\,\overline{C}_2\right)\left(1 \pm \left[\frac{\Delta C_1}{C_1} + \frac{\Delta C_2}{C_2}\right]\right)}{\left(\overline{C}_1 + \overline{C}_2 \pm \left[\Delta C_1 + \Delta C_2\right]\right)}$$

Convert the uncertainty in the denominator to the relative form and apply U2 to the division.

$$C_s \pm \Delta C_s = \left[\frac{\overline{C}_1\,\overline{C}_2}{\overline{C}_1 + \overline{C}_2}\right]\left[1 \pm \left[\frac{\Delta C_1}{C_1} + \frac{\Delta C_2}{C_2} + \frac{\Delta C_1 + \Delta C_2}{C_1 + C_2}\right]\right]$$

Do measured values and predicted values agree ? Ask yourself the question: Do the ranges of values measured ± uncertainty and predicted ± uncertainty overlap ? If they overlap comfortably, you report that the value predicted agrees with the measured value to within the accuracy of the experiment. If the value ranges barely overlap, you report that the results suggest the formula predicts the value of the series combination, but that the result is more uncertain. If the value ranges barely fail to overlap, you report that the results may suggest the formula predicts the value of the series combination, but that the result is not supported by the current set of measurements. If the two ranges fail to overlap, you report that the results suggest the formula does not predict the value of the series combination to the accuracy of the experiment. **In all cases**, you include the measured and predicted values with their uncertainties in the final summary report.

Addition of Capacitors in Parallel:

Derive the uncertainty propagation formula for capacitors in parallel.

$$C_p = C_1 + C_2 \to \overline{C}_p \pm \Delta C_p = ???$$

Discuss carefully the degree to which your measurements support the rule for the addition of capacitors in parallel.

METER UNCERTAINIES & UNCERTAINTY PROPAGATION

Exercise 1: Consider the case that $\tau = {}^{1}/_{RC}$. The resistance read is $(39.2\pm0.2)\Omega$, and the capacitance read is $(0.025 \pm 0.001)\ mF$. Compute τ with its uncertainty.

Exercise 2: The 200 Ω resistance scale is the most sensitive for the Mastech, and it has a resolution of 0.01Ω and uncertainty specs of 0.5% rdg ± 10 digits. Two resistors are tested leading to measured values of 39.2 Ω and 21.9 Ω. Compute the uncertainties for each measured value and report the results in the $\overline{R} \pm \Delta R$ form. Compute the values expected for the series and parallel combinations of these resistors propagating the uncertainties to yield predicted values $\overline{R}_S \pm \Delta R_S$ and $\overline{R_{||}} \pm \Delta R_{||}$. The resistance of the parallel combination is measured to be 14.1 Ω. What is the uncertainty for this result value? Do the predicted and measured values of the parallel combination agree ? The measured value for the series combination is 60.2 Ω. What is the uncertainty for this result value? Do the predicted and measured values of the series combination agree ? It is noted that shorting the meter test leads gives a meter reading of 0.3 Ω. Might this explain the apparent disagreement ?

The uncertainty in a measured value is an estimate of the half-width of the range about the most probable value in which the actual value most likely falls. That uncertainty has contributions that are statistical (due to the randomness of some measurements), systematic (due to some recurring offset or calibration error) and procedure (you made a mistake) contributions.

MOST PROBABLE VALUE AND STATISTICAL UNCERTAINTIES If a sequence of repeated measurements of the same physical quantity yields a set of N values $\{x_1, x_2 \ldots x_N\}$, then the most probable value for the quantity is the arithmetic mean of the values.

$$\text{MOST PROBABLE VALUE:} \quad \overline{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

We use the standard deviation as our estimate of the statistical uncertainty: $\delta x_{st} = \sigma$

$$\text{STANDARD DEVIATION:} \quad \sigma = \delta\mathbf{x} = \sqrt{\frac{1}{(N-1)}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

The standard deviation (our estimate of the random reading fluctuation contribution)is not the uncertainty in the measurement. It is one contribution to the uncertainty. Systematic uncertainty contributions must be added to the standard deviation.

Normal Gaussian Error Curve: mean value x-bar and standard deviation σ.

We expect that 68% of the values fall within ±σ of the mean value of x-bar, 95% within ± 2 σ of the mean, and 99.7% within ± 3 σ of the mean.

METER UNCERTAINIES & UNCERTAINTY PROPAGATION

ABSOLUTE & RELATIVE UNCERTAINTIES: In the expression $x \pm \delta x$, δx is the absolute uncertainty for the value x. We can rewrite this as $x\,(1 \pm {}^{\delta x}/_{x})$. We will refer to ${}^{\delta x}/_{x}$ as the relative (fractional) uncertainty in x. It follows that when you add or subtract quantities, you add their absolute uncertainties. When you multiply or divide quantities, you add their relative uncertainties. As an example for $V = \pi r^2 H$, we would compute the relative uncertainty $\delta V/V$ as $2\,(\delta r/r) + \delta H/H$. As we refine our methods, we will learn that independent uncertainties should be added in quadrature. That is: If the measurements of H and r are assumed to be independent of one another, we should estimate the relative uncertainty in the volume as:

$$\frac{\delta \mathbf{V}}{\mathbf{V}} = \sqrt{\left(2\frac{\delta \mathbf{r}}{\mathbf{r}}\right)^2 + \left(\frac{\delta \mathbf{H}}{\mathbf{H}}\right)^2}$$

RELATION TO TOTAL DIFFERENTIAL: If we have a function of several independent variables, say f(x,y,z), then we can define the total differential of f as

$$df = f(x+dx,y+dy,z+dz) - f(x,y,z)$$

where dx, dy, and dz are small. The total differential may be computed as

$$\mathbf{df} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\,\mathbf{dx} + \frac{\partial \mathbf{f}}{\partial \mathbf{y}}\,\mathbf{dy} + \frac{\partial \mathbf{f}}{\partial \mathbf{z}}\,\mathbf{dz}$$

Where $\partial f/\partial x$ is the partial derivative of f with respect to x. This partial is computed by finding the derivative of f with respect to x given that y and z are treated as constants.
Example: Let $f(x,y) = x^2 y$. We find $df = f(x+dx,y+dy) - f(x,y) = 2\,x\,y\,dx + x^2\,dy$.
A numeric example: $f(3.01,1.995) - f(3,2) \approx 2(3)(2)\,(.01) + 3^2\,(-.005)$

$$18.07490 - 18.0 \approx .12 - .045 = .0075$$

The values differ only by terms of order $(dx)^2$, $(dy)^2$ or smaller. We have used first derivatives to get a first order estimate of the change in the value of the function. Terms that are quadratic or higher order in small things (dx & dy) have been neglected.
USING PARTIAL DERIVATIVE TO PROPAGATE UNCERTAINTIES: Suppose that we measured the radius (or diameter) and height of a cylinder and determined the associated uncertainties. How could we estimate the uncertainty in the volume that is to be calculated using these measured values ? The volume of the cylinder may be computed as $V = \pi r^2 H$. It follows that:

$$dV = \frac{\partial V}{\partial H}\,dH + \frac{\partial V}{\partial r}\,dr = \pi\, r^2\, dH + H\, 2\pi\, r\, dr$$

When we wish to compute the uncertainty δV for the volume, it seems reasonable that we use the partial derivative, but substitute the uncertainties in the measurements for the differentials of r and H.

$$\delta V = \frac{\partial V}{\partial H}\,(\pm\delta H) + \frac{\partial V}{\partial r}\,(\pm\delta r) = \pi\, r^2\,(\pm\delta H) + H\, 2\pi\, r\,(\pm\delta r)$$

Realizing that errors are very clever and conspire to degrade our measurements, we must use the absolute values of the terms.

METER UNCERTAINIES & UNCERTAINTY PROPAGATION

$$\delta V = \left|\frac{\partial V}{\partial H}\ \delta H\right| + \left|\frac{\partial V}{\partial r}\ \delta r\right| = \left|\pi r^2\ \delta H\right| + \left|H\ 2\pi r\ \delta r\right|$$

$$\frac{\delta V}{V} = \frac{\pi r^2\ (\delta H)}{\pi r^2\ H} + \frac{H\ 2\pi r\ (\delta r)}{\pi r^2\ H} = \frac{\delta H}{H} + 2\ \frac{\delta r}{r}$$

Compare with the result obtained by adding relative errors.

If we believe the measurement errors in r and H to be independent of one another, we would add them in quadrature.

$$\delta V = \sqrt{\left|\frac{\partial V}{\partial H}\ \delta H\right|^2 + \left|\frac{\partial V}{\partial r}\ \delta r\right|^2} = \sqrt{(\pi r^2\ \delta H)^2 + (H\ 2\pi r\ \delta r)^2}$$

Exercise: Develop the corresponding expression for the uncertainty in the acceleration due to gravity δg given that g is to be computed as $g = {}^{2H}/_{t^2}$ where the measured values H and t have uncertainties δH and δt.

The Rules: In SP211/2 we rarely add in quadrature, we just add absolute uncertainties if quantities are added or subtracted, add relative uncertainties if quantities are multiplied or divided and multiply relative uncertainties by the power when values are raised to a power.

Add currents: $I_1 + I_2 = (0.062 \pm 0.002)\,A + (0.037 \pm 0.003)\,A = (0.099 \pm 0.005)\,A$

add absolute uncertainties; do the same when subtracting values !

Power:
$$P = I\,V = (0.062 \pm 0.002)\,A * (1.52 \pm 0.03)\,V$$
$$= 0.062\ (1 \pm 0.032)\,A * 1.52\ (1 \pm 0.02)\,V = 0.094\,(1 \pm 0.05)\ W$$

add relative uncertainties

More Power: $P = \dfrac{V^2}{R} = \dfrac{[1.52\ (1 \pm 0.02)\,V]^2}{22\ (1 \pm 0.03)\,\Omega} = 0.105\ (1 \pm 0.07)\ W$

division => also add the relative uncertainty

Just a Power: $V^2 = [1.52\ (1 \pm 0.02)\,V]^2 = 2.31\left(1 \pm [2 * 0.02]\right)V^2$

multiply relative uncertainty by the power

An absolute error is the direct statement of the uncertainty range. A value expected to be 5 that ranges between 4 and 6 is 5 ± 1 where 1 is the absolute error. The relative error is the fractional uncertainty. Hence 5 ± 1 can be restated as $5\ (1 \pm 0.2)$ where 0.2 is the fractional or relative error.

Exercise 3: Develop the an expression for the uncertainty of a sum $f(A,B) = A + B$ using the partial derivative approach. $A = \overline{A} \pm \delta A;\ B = \overline{B} \pm \delta B$

Exercise 4: Develop the an expression for the uncertainty of a ratio $f(A,B) = {}^{A}/_{B}$ using the partial derivative approach. $A = \overline{A} \pm \delta A;\ B = \overline{B} \pm \delta B$

Exercise 5: Develop the an expression for the uncertainty of a power $f(A,n) = A^{\,n}$ using the partial derivative approach. $A = \overline{A} \pm \delta A;\ n \equiv n$